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DETERMINATION OF THE IMPLEMENTATION OF THE 3-AXIS  
ATTITUDE MOTION SIMULATOR DIGITAL POSITION CONTROLLER

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MOTION SIMULATOR DIGITAL POSITION CONTROLLER

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ABSTRACT

In this work we mathematically reconstruct and document the digital position controller implemented in the control computer of the 3-axis attitude motion simulator, since the information supplied with the executable code of this controller was insufficient to make substantial modifications to it. We also develop methodologies to introduce changes in the controller which do not require rewriting the software. Finally, recommendations are made on possible improvement to the control system performance.

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## INTRODUCTION

The objective of this project is to investigate the nature and details of the implementation of the digital position controller of the three-axes attitude motion simulator in order to effect changes in its parameters without having to rewrite the software, since the source code is not available.

To get a good understanding of this digital position controller, several experiments were carried out on the simulator via the control computer, with the assistance of NASA/Boeing personnel. The goal of these experiments was to determine whether or not the position controller implemented in the control computer was the discrete-time equivalent of the position controller implemented for the analog mode operation of the simulator [1].

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# DETERMINATION OF THE DIGITAL POSITION CONTROLLER

During the analog mode operation of the simulator, the position controller of any of the three axes has the form

$$G_c(s) \triangleq \frac{U(s)}{E(s)} = \frac{K(s/\omega_1+1)(s/\omega_2+1)}{s(s/\omega_3+1)}, \quad (1)$$

where  $E(s)$  is the Laplace transform of the error signal  $e(t)$  and  $U(s)$  is the Laplace transform of the control signal  $u(t)$ . The error signal  $e(t)$  is defined as the difference between the commanded angular position  $\theta_c(t)$  and the actual system angular position  $\theta(t)$ .

To obtain a discrete-time equivalent of this analog controller, the industry-accepted way of doing it is by applying the bilinear transformation, since it preserves the stability properties of its continuous-time counterpart, i.e.,

$$\begin{aligned} G_c(z) &\triangleq \frac{U(z)}{E(z)} \triangleq G_c(s) \Big|_s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}, \\ &= \frac{K \left( \frac{2}{T\omega_1} \frac{1-z^{-1}}{1+z^{-1}} + 1 \right) \left( \frac{2}{T\omega_2} \frac{1-z^{-1}}{1+z^{-1}} + 1 \right)}{\left( \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} \right) \left( \frac{2}{T\omega_3} \frac{1-z^{-1}}{1+z^{-1}} + 1 \right)}, \\ &= K_0 \left( \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + a_2 z^{-2}} \right), \end{aligned} \quad (2)$$

where

$E(z) \triangleq$  Z-transform of the discrete-time error signal  $e(kT)$ ,

$U(z) \triangleq$  Z-transform of the discrete-time control signal  $u(kT)$ ,

$$K_0 \triangleq \frac{K \omega_3}{\omega_1 \omega_2}, \quad (3)$$

$$a_0 \triangleq \frac{\left(\frac{2}{T}\right)^2 + \frac{2}{T}(\omega_1 + \omega_2) + \omega_1 \omega_2}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} \omega_3}, \quad (4)$$

$$a_1 \triangleq - \frac{2\left[\left(\frac{2}{T}\right)^2 - \omega_1 \omega_2\right]}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} \omega_3}, \quad (5)$$

$$a_2 \triangleq \frac{\left(\frac{2}{T}\right)^2 - \frac{2}{T}(\omega_1 + \omega_2) + \omega_1 \omega_2}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} \omega_3}, \quad (6)$$

$$b_1 \triangleq \frac{\frac{4}{T} \omega_3}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} \omega_3}, \quad (7)$$

$$b_2 \triangleq - \frac{\left(\frac{2}{T}\right)^2 - \frac{2}{T} \omega_3}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} \omega_3}, \quad (8)$$

$T \triangleq$  sampling interval (in seconds).

From equation (2) we find that the output of the digital position controller at the sampling instant  $kT$  is given by

$$\begin{aligned} u(kT) = & -b_1 u(kT-T) - b_2 u(kT-2T) + K_0 a_0 e(kT) + K_0 a_1 e(kT-T) \\ & + K_0 a_2 e(kT-2T). \end{aligned} \quad (9)$$

According to the information available on the implementation of the digital position control algorithm [1], the parameters of the controller can be changed from the keyboard when INIT (initialization) is selected from the control computer main menu screen. When this entry is selected, a total of nine constants (labeled "coefficients") per axis appear on the screen; however, it is known that only seven of them correspond to the digital position control algorithm, one to the digital rate command scaling, and one is set to zero.

As shown in equation (9), there is a total of five coefficients, which strongly suggests that the digital position controller implemented in the control computer is not the one obtained by applying the bilinear transform to the analog position controller transfer function given by equation (1).

It should be pointed out at this stage, nonetheless, that there are other realizations of the digital position controller given by equation (2) and that more constants could be generated for scaling purposes to avoid the usual problems encountered when fixed point arithmetic is used. However, the numerical values of the coefficients of these other realizations do not match those that appear on the INIT screen for any of the three axes.

To figure out the actual structure of the controller, several experiments were carried out to determine, for example, if integration of the error signal was taking place to make sure that the controller integrator had been implemented. Experimental results indicated that integration of the input signal did indeed occur, therefore, to generate the exact seven coefficients per axis, it became apparent that either a pole or a pole and a zero had been added to the original analog position controller.

Let us modify the analog position controller by adding a pole at  $s = -\omega_4$ , i.e.,

$$G'_c(s) = K \frac{(s/\omega_1 + 1)(s/\omega_2 + 1)}{s(s/\omega_3 + 1)(s/\omega_4 + 1)} \quad (10)$$

Then, application of the bilinear transform to this controller transfer function yields the following discrete-time equivalent controller

$$\begin{aligned} G'_c(z) &= G'_c(s) \Big|_{s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}} , \\ &= K'_0 \left( \frac{c_0 + c_1 z^{-1} + c_2 z^{-2} + c_3 z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}} \right) , \end{aligned} \quad (11)$$

where

$$K'_0 = \frac{KT\omega_3\omega_4}{2\omega_1\omega_2} \quad (12)$$



$$c_0 = \frac{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_1 + \omega_2) + \omega_1 \omega_2}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} , \quad (13)$$

$$c_1 = \frac{-\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_1 + \omega_2) + 3\omega_1 \omega_2}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} , \quad (14)$$

$$c_2 = \frac{-\left(\frac{2}{T}\right)^2 - \frac{2}{T} (\omega_1 + \omega_2) + 3\omega_1 \omega_2}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} , \quad (15)$$

$$c_3 = \frac{\left(\frac{2}{T}\right)^2 - \frac{2}{T} (\omega_1 + \omega_2) + \omega_1 \omega_2}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} , \quad (16)$$

$$d_1 = \frac{-3\left(\frac{2}{T}\right)^2 - \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} , \quad (17)$$

$$d_2 = \frac{3\left(\frac{2}{T}\right)^2 - \frac{2}{T} (\omega_3 + \omega_4) - \omega_3 \omega_4}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} , \quad (18)$$

$$d_3 = \frac{-\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) - \omega_3 \omega_4}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4} . \quad (19)$$

A direct time domain realization of this digital controller is given by

$$u(kT) = -d_1 u(kT-T) - d_2 u(kT-2T) - d_3 u(kT-3T) + K'_0 c_0 e(kT) \\ + K'_0 c_1 e(kT-T) + K'_0 c_2 e(kT-2T) + K'_0 c_3 e(kT-3T) \quad . \quad (20)$$

This direct realization of the digital position controller is by no means the best realization as it is susceptible to round-off and quantization errors. However, the numerical values of its coefficients were found to be the same as those that appear on the INIT screen for each of the three axes once  $\omega_4$  was determined, thus leading to the conclusion that the controller given by equations (11) and (20) is the one that has been implemented in the control computer (PC). In fact, the extra pole ( $-\omega_4$ ) is located at -20, -25, and -22.8 for the roll, yaw, and sidereal axis, respectively.

The following analog transfer functions were determined to have been the source of the discrete-time, equivalent position controllers implemented in the control computer.

Roll:

$$G'_c(s) = 0.0047 \frac{(s/0.056 + 1)(s/0.66 + 1)}{s(s/5.6 + 1)(s/20 + 1)} \quad , \quad (21)$$

Yaw:

$$G'_c(s) = 0.025 \frac{(s/0.095 + 1)(s/0.588 + 1)}{s(s/1.43 + 1)(s/25 + 1)} \quad , \quad (22)$$

Sidereal:

$$G'_c(s) = 0.03 \frac{(s/0.068 + 1)(s/12.5 + 1)}{s(s/20 + 1)(s/22.8 + 1)} \quad . \quad (23)$$

Comparing equations (21) to (23) with those found on page 3-13 of Reference 1, we can conclude that the analog position controllers were modified by adding a pole before they were discretized to obtain the digital position controllers.

Application of the bilinear transform (with  $T = 0.01$  sec) to equations (21) to (23) yields the following discrete-time equivalent position controllers.

Roll:

$$G'_c(z) = \frac{6.32005 \times 10^{-2} - 6.274936 \times 10^{-2} z^{-1} - 6.320026 \times 10^{-2} z^{-2} + 6.274959 \times 10^{-2} z^{-3}}{1 - 2.763707 z^{-1} + 2.537319 z^{-2} - 7.736116 \times 10^{-1} z^{-3}} \quad , \quad (24)$$

Yaw:

$$G'_c(z) = \frac{7.08467 \times 10^{-2} - 7.036407 \times 10^{-2} z^{-1} - 7.084631 \times 10^{-2} z^{-2} + 7.036446 \times 10^{-2} z^{-3}}{1 - 2.763579 z^{-1} + 2.530314 z^{-2} - 7.667345 \times 10^{-1} z^{-3}}, \quad (25)$$

Sidereal:

$$G'_c(z) = \frac{6.979685 \times 10^{-2} - 6.153802 \times 10^{-2} z^{-1} - 6.979127 \times 10^{-2} z^{-2} + 6.154359 \times 10^{-2} z^{-3}}{1 - 2.613514 z^{-1} + 2.26424 z^{-2} - 6.507263 \times 10^{-1} z^{-3}}. \quad (26)$$

Let

$$K_1 \triangleq K'_0 c_0, \quad (27)$$

$$K_2 \triangleq K'_0 c_1, \quad (28)$$

$$K_3 \triangleq K'_0 c_2, \quad (29)$$

$$K_4 \triangleq K'_0 c_3, \quad (30)$$

$$K_5 \triangleq -d_1, \quad (31)$$

$$K_6 \triangleq -d_2, \quad (32)$$

$$K_7 \triangleq -d_3. \quad (33)$$

Then the digital position controller for any of the three axes is given by

$$G'_c(z) = \frac{K_1 + K_2 z^{-1} + K_3 z^{-2} + K_4 z^{-3}}{1 - K_5 z^{-1} - K_6 z^{-2} - K_7 z^{-3}}. \quad (34)$$

The reason for writing the digital position controller in the form of equation (34) is that when the INIT menu selection is made, the coefficients of the digital position controller appear on the control computer monitor screen as  $K_1$  through  $K_7$ . For example, the coefficients of the digital position controller for the roll axis are:

$$K_1 = 6.32005 \times 10^{-2} ,$$

$$K_2 = -6.274936 \times 10^{-2} ,$$

$$K_3 = -6.320026 \times 10^{-2} ,$$

$$K_4 = 6.274959 \times 10^{-2} ,$$

$$K_5 = 2.763707 ,$$

$$K_6 = -2.537319 ,$$

$$K_7 = 7.736116 \times 10^{-1} .$$

The present implementation of the digital position controller is therefore given by

$$\begin{aligned} u(kT) = & K_5 u(kT-T) + K_6 u(kT-2T) + K_7 u(kT-3T) + K_1 e(kT) + K_2 e(kT-T) \\ & + K_3 e(kT-2T) + K_4 e(kT-3T) . \end{aligned} \quad (35)$$

Remark: The dc gains of the controllers given by equations (21) through (23) are smaller than those of the original analog position controllers (shown on page 3-13 of Reference 1) because of the fact that the position sensors used in the analog and digital modes are different. In the analog mode, a single turn potentiometer is used to sense the angular position. This potentiometer has a gain embedded in it, in fact, it is equal to 0.05 Volts/degree for the roll axis and 0.1 Volts/degree for the yaw and sidereal axis. In the digital mode, a resolver-inductosyn pair, along with two resolver to digital converters, are used to obtain the angular displacement. The resolver to digital converters have a third-order dynamic model with unity dc gain. Therefore, this change in the gain is reflected by the reduction of the gain of the modified analog equivalent position controllers of equations (21) through (23).

## CONCLUSIONS AND RECOMMENDATIONS

The structure of the three-axes attitude motion simulator digital position controller implemented in the control computer has been determined through experimentation and educated guesses, since no documentation about (or computer program source code implementation of) such a controller is available.

Thus, given that only the executable code of the controller is available, its structure cannot be changed without rewriting the entire control software; however, it is possible to make some on-line changes which do not require new controller software. For instance, the original location of the analog equivalent position controller poles and zeros could be changed to new desired values  $-\omega'_1$ ,  $-\omega'_2$ ,  $-\omega'_3$ , and  $-\omega'_4$  and then compute  $K_1$  through  $K_7$  off-line using equations (27) to (33). Also, another zero located at  $s = -\omega_0$  could be added to improve global stability as well as the speed of response of the simulator.

If a zero is added to the modified analog equivalent position controller, i.e.,

$$G'_c(s) = K' \frac{(s/\omega_0 + 1)(s/\omega_1 + 1)(s/\omega_2 + 1)}{s(s/\omega_3 + 1)(s/\omega_4 + 1)}, \quad (36)$$

then the discrete-time equivalent position controller will still be given by equation (11), however,  $K'_0$ ,  $c_0$ ,  $c_1$ ,  $c_2$ , and  $c_3$  will change to

$$K'_0 = \frac{K' T \omega_3 \omega_4}{2 \omega_0 \omega_1 \omega_2}, \quad (37)$$

$$c_0 = \frac{\left(\omega_0 + \frac{2}{T}\right) \left[ \left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_1 + \omega_2) + \omega_1 \omega_2 \right]}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4}, \quad (38)$$

$$c_1 = \frac{\left(\omega_0 + \frac{2}{T}\right) \left[ -2 \left(\frac{2}{T}\right)^2 + 2 \omega_1 \omega_2 \right] + \left(\omega_0 - \frac{2}{T}\right) \left[ \left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_2 + \omega_2) + \omega_1 \omega_2 \right]}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4}, \quad (39)$$

$$c_2 = \frac{\left(\omega_0 + \frac{2}{T}\right) \left[\left(\frac{2}{T}\right)^2 - \frac{2}{T} (\omega_1 + \omega_2) + \omega_1 \omega_2\right] + \left(\omega_0 - \frac{2}{T}\right) \left[-2 \left(\frac{2}{T}\right)^2 + 2\omega_1 \omega_2\right]}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4}, \quad (40)$$

$$c_3 = \frac{\left(\omega_0 - \frac{2}{T}\right) \left[\left(\frac{2}{T}\right)^2 - \frac{2}{T} (\omega_1 + \omega_2) + \omega_1 \omega_2\right]}{\left(\frac{2}{T}\right)^2 + \frac{2}{T} (\omega_3 + \omega_4) + \omega_3 \omega_4}. \quad (41)$$

The new values of the coefficients  $K_1$  to  $K_4$  can again be computed off-line using equations (37) to (41) and (27) to (30). Notice that the values of the coefficients  $K_5$  to  $K_7$  will not change because they do not depend on the zeros. It should also be pointed out that when a zero is added to the modified analog equivalent position controller the value of the coefficients  $K_1$  through  $K_4$  may be larger than 1 and can cause some overflow problems [5] if provisions are not made to handle such numbers because the INTEL 8086 microprocessor utilizes 2's complement arithmetic. This problem can be alleviated by reducing the dc gain  $K'$  of the equivalent analog position controller of equation (36).

Another possible improvement in the performance of the simulator could be achieved by introducing frequency prewarping of the analog equivalent position controller corner frequencies; however, the improvement would be minute in this case because the sampling rate used in the control algorithm is unusually high (100 Hz).

Once the coefficients  $K_1$  through  $K_7$  have been recomputed off-line, the old values can be changed interactively in the control computer when INIT is selected from the main menu.

It should be apparent by now that the suggested changes only involve the recomputation of the coefficients  $K_1$  through  $K_7$  since they reflect any changes that may take place in the location of the poles and zeros of the modified analog equivalent position controller of equation (10) or the addition of another zero as in equation (36).

Although the performance requirements of the simulator are not terribly stringent, it is deemed by the author that a much more robust controller can be designed to operate at a substantially lower sampling rate (much lower than 100 Hz as it is presently implemented) and yet outperform the present digital position controller in every aspect, such as global stability, speed of response, and flexibility. This, however, would require rewriting the control software.

To begin with, an improved controller design would have to take into account the dynamics of the resolver to digital converter which is used to determine the angular position of each of the simulator axes, since this device is installed in the feedback path of the position loop. This type of converter is modeled as a type II tracking loop and possesses the closed-loop transfer function (ILC Data Devices Corp.)

$$G_{R/D}(s) = \frac{A(s/B + 1)}{\frac{1}{10B} s^3 + s^2 + \frac{A}{B} s + A}, \quad (42)$$

where

$$A = \begin{cases} 50176 & , \text{ XDC-19147-303} \\ 184800 & , \text{ XDC-19197-304} \end{cases},$$

$$B = \begin{cases} 100 & , \text{ XDC-19147-303} \\ 300 & , \text{ XDC-19197-304} \end{cases},$$

where the XDC-19147-303 and the XDC-19197-304 are the 14- and 16-bit resolution resolver to digital converters installed in the attitude simulator digital control cards, respectively. Furthermore, it would make use of the velocity signal which is also generated by the same resolver to digital converter since the noise content of this signal is much smaller than the one generated by a mechanical transducer such as a tachometer.

This new controller would also implement the rate compensation totally in the digital domain since the velocity signal generated by the resolver to digital converter can be digitized using an analog to digital converter without much difficulty. This approach would add a tremendous flexibility to the controller design.

Finally, this controller would be designed in such a way that its performance does not deteriorate with changes of simulator parameters such as inertia, i.e., it would have some level of adaptation that takes care of variations in the system parameters.

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